Directions of point sets in affine planes

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An *affine plane* of order q is a $2-(q^2, q, 1)$ design. The classical construction, denoted $\mathrm{AG}(2, q)$, has a vector space $\mathbb{F}_q^2$ as point set, and the affine lines of the space as blocks (or *lines*). The lines come in $q+1$ parallel classes, and to each parallel class we assign a *slope* or *direction*. A set S of q points is said to *determine* a direction d if some line with slope d is spanned by two points of S. Note that d is **not** determined by S if and only if every line with slope d intersects S in exactly one point. A classical result states that a set S of q points in $\mathrm{AG}(2, q)$ that determines at most $\frac{q+1}{2}$ directions must be a translate of a vector subspace of $\mathbb{F}_q^2$ over some subfield of $\mathbb{F}_q$.

Recently, the problem was generalized to study sets of kq points for some integer k. We say that S is *equidistributed* from direction d if all lines with slope d intersect S in k points, and we call d a *special* direction otherwise. Surprisingly, every translation plane (which includes all planes $\mathrm{AG}(2, q)$) has a set with exactly 3 special directions.

In this talk, I will discuss some of these new result, and some other generalizations of the classical problem of determined directions. This is based on joint work with Bence Csajbók, Tamás Szőnyi, and Zsuzsa Weiner [1, 2]

References

[1] Sam Adriaensen, Tamás Szőnyi, Zsuzsa Weiner *Multisets with few special directions and small weight codewords in Desarguesian planes*, Des. Codes Cryptogr., to appear.

[2] Sam Adriaensen, Zsuzsa Weiner *Points below a parabola in affine planes of prime order*, Bull. Belg. Math. Soc. Simon Stevin, Vol.32, No. 3, August 2025, 332-342.