On latin-square graphs avoiding K33

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We discuss the problem of existence of latin squares without a substructure consisting of six elements (r_1, c_2, l_3), (r_2, c_3, l_1), (r_3, c_1, l_2), (r_2, c_1, l_3), (r_3, c_2, l_1), (r_1, c_3, l_2), where (r, c, l) means that the cell in the rth row and cth column contains the symbol l.

·	**A**	**B**
A	·	**C**
B	**C**	·

Equivalently, the corresponding latin square graph does not have an induced subgraph isomorphic to $K_{3,3}$. The exhaustive search [1] says that there are no such latin squares of order from 3 to 11, and there are only two $K_{3,3}$-free latin squares of order 8, up to equivalence. We repeat the search, establishing also the number of latin m-by-n rectangles for each m and n less or equal to 11. As a switched combination of two orthogonal latin squares of order 8, we construct a $K_{3,3}$-free (universally noncommutative) latin square of order 16.

1	0	2	5	6	11	4	13	15	8	10	9	12	3	14	7
0	1	5	2	11	6	13	4	8	15	9	10	3	12	7	14
2	3	7	0	4	9	6	15	10	13	11	8	14	1	12	5
3	2	0	7	9	4	15	6	13	10	8	11	1	14	5	12
4	5	10	1	0	15	9	14	2	11	12	13	8	7	3	6
5	4	1	10	15	0	14	9	11	2	13	12	7	8	6	3
10	7	4	3	13	14	0	11	12	9	2	15	5	6	8	1
7	10	3	4	14	13	11	0	9	12	15	2	6	5	1	8
8	9	15	14	12	7	10	3	0	5	1	6	2	13	4	11
9	8	14	15	7	12	3	10	5	0	6	1	13	2	11	4
11	14	8	13	10	5	12	1	7	6	0	3	4	15	2	9
14	11	13	8	5	10	1	12	6	7	3	0	15	4	9	2
6	13	12	11	3	8	2	7	4	1	14	5	9	0	10	15
13	6	11	12	8	3	7	2	1	4	5	14	0	9	15	10
12	15	6	9	2	1	5	8	14	3	4	7	10	11	13	0
15	12	9	6	1	2	8	5	3	14	7	4	11	10	0	13

The problem can be generalized to the study of $K_{k+2,k+2}$-free collections of k mutually orthogonal latin squares. For example, among the two linear pairs of orthogonal latin squares over GF(7), one is $K_{4,4}$-free and the other is not.

This is joint work with Aleksandr Krotov.

References

[1] A. Brouwer, I. M. Wanless, *Universally noncommutative loops*, Bull. Inst. Comb. Appl. 61, 2011, 113вЂ“-115.