On strong nodal domains for eigenfunctions of Hamming graphs

Alexandr Valyuzhenich
Hebei Normal University
graphkiper@mail.ru

The Laplacian matrix of the n-dimensional hypercube has $n+1$ distinct eigenvalues $2i$, where $0 \leq i \leq n$. In 2004, Bıyıkoğlu, Hordijk, Leydold, Pisanski and Stadler [1] initiated the study of eigenfunctions of hypercubes with the minimum number of weak and strong nodal domains. In particular, they proved that for every $1 \leq i \leq \frac{n}{2}$ there is an eigenfunction of the hypercube with eigenvalue $2i$ that have exactly two strong nodal domains. Based on computational experiments, they conjectured that the result also holds for all $1 \leq i \leq n-2$. In this work, we confirm their conjecture for $i \leq \frac{2}{3}(n-\frac{1}{2})$ if i is odd and for $i \leq \frac{2}{3}(n-1)$ if i is even. We also consider this problem for the Hamming graph $H(n,q)$, $q \geq 3$ (for $q=2$, this graph coincides with the n-dimensional hypercube), and obtain even stronger results for all $q \geq 3$.

This is a joint work with Konstantin Vorob'ev.

References

[1] T. Bıyıkoğlu, W. Hordijk, J. Leydold, T. Pisanski, P. F. Stadler, *Graph Laplacians, nodal domain and hyperplane arrangements*, Linear Algebra and its Applications 390 (2004) 155–174.