New infinite families of divisible design graphs, which are covers of strongly regular polar graphs

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We consider only simple graphs. A *divisible design graph* with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ is a k-regular graph on v vertices such that its vertex set can be partitioned into m classes of size n where any two distinct vertices from the same class have exactly λ_1 common neighbours and any two vertices from different classes have exactly λ_2 common neighbours. Divisible design graphs were introduced in [3] as a bridge between graph theory and the theory of group divisible designs. Since then, tens of constructions of divisible design graphs have been introduced.

In [1, Proposition 12.5.3], a construction of antipodal distance-regular graphs of diameter 3 was given. This construction uses a vector space of dimension 2 equipped with a nondegenerate symplectic bilinear form. Note that the construction admits the vector spaces over all finite fields $\mathbb{F}_q$. Another ingredient of the construction above is a subgroup N of index $r \geq 2$ in $\mathbb{F}_q^*$. In our work we show that it is possible to slightly modify this construction by plugging a difference set in the (cyclic) quotient group $\mathbb{F}_q^+/N$ into it and letting the dimension of the vector space be an arbitrary positive even integer. This leads to a new infinite family of divisible design graphs that are r-covers of the corresponding symplectic strongly regular polar graphs [2, Section 2.5].

Further, we mimic the construction of divisible design graphs above by replacing the symplectic bilinear form with a bilinear (resp. sesquilinear) form obtained by the polarisation of hyperbolic, elliptic and parabolic quadratic forms (resp. by the polarisation of the Hermitian form). This gives a few more infinite families of r-covers of strongly regular polar graphs (see [2, Section 2.6, Section 2.7]). These infinite families of r-covers contain infinite subfamilies of non-trivial divisible design graphs in the elliptic, parabolic and Hermitian cases.

This is joint work with Bart De Bruyn and Sergey Goryainov.

References

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