Feasible parameters of non-trivial divisible design graphs

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September 3, 2025

1 Algorithm for generating feasible parameters of non-trivial DDGs

- We exclude all prime v.
- We exclude all tuples for which v is odd and k is odd.
- We exclude all tuples for which $k \leq 2$ or $k \geq v-2$.
- We exclude all tuples for which $\lambda_1 < \mathrm{Max}(0, 2k-v)$ or $\lambda_2 < \mathrm{Max}(0, 2k-v)$.
- We exclude all tuples for which $\lambda_1 = \lambda_2$.
- For a DDG with parameters $(v, k, \lambda_1, \lambda_2, m, n)$, we always have $k^2 = k + \lambda_1(n-1) + \lambda_2 n(m-1)$; see [3].
- DDGs with $\lambda_2 = 0$ are characterised in [3, Proposition 4.3]; we exclude such tuples.
- DDGs with $\lambda_2 = 2k-v$ are characterised in [3, Proposition 4.7]; we exclude such tuples.
- DDGs with $\lambda_1 = k$ are characterised in [3, Proposition 4.5]; we exclude such tuples.
- Deza graphs with $k = b+1$ are characterised in [2] and [4]; we thus exclude tuples for which $\lambda_1 = k-1$ or $\lambda_2 = k-1$.
- If $k-\lambda_1$ is not a square and $k\lambda_2$ is odd, we exclude such tuples due to [1, Theorem 3.6].
- If $\lambda_1 = 0$, we exclude the tuples for which $nk > v$ and the tuples for which $nk = v$ and $k \neq n\lambda_2$.
- We exclude tuples for which $k-\lambda_1$ is not square and $k^2-\lambda_2 v$ is not a square.
- We exclude tuples for which $k-\lambda_1$ is not square and $m(n-1)$ is not divisible by 2.
- We exclude tuples for which $k^2-\lambda_2 v$ is not a square and $m-1$ is not divisible by 2.
- We exclude tuples for which $k-\lambda_1 = k^2-\lambda_2 v$.
- We exclude tuples for which $k^2-\lambda_2 v < 0$.
- If $k-\lambda_1$ is a square and $k^2-\lambda_2 v$ is not a square, we exclude tuples for which k is not divisible by $\sqrt{k-\lambda_1}$ and for which k is divisible by $\sqrt{k-\lambda_1}$ and $m(n-1) - k/\sqrt{k-\lambda_1}$ is not divisible by 2. Otherwise the multiplicities f_1, f_2, g_1, g_2 are uniquely determined.
- If $k-\lambda_1$ is not square and $k^2-\lambda_2 v$ is a square, we exclude tuples for which $k^2-\lambda_2 v \neq 0$ and k is not divisible by $\sqrt{k^2-\lambda_2 v}$ and for which $k^2-\lambda_2 v \neq 0$, k divisible by $\sqrt{k^2-\lambda_2 v}$ and $m-1-k/\sqrt{k^2-\lambda_2 v}$ is not divisible by 2. Otherwise the multiplicities f_1, f_2, g_1, g_2 are uniquely determined.
- If $k-\lambda_1$ is a square and $k^2-\lambda_2 v = 0$, the multiplicities are f_1, f_2, g_1, g_2 are uniquely determined, where we put $g_1 = m-1$ and $g_2 = 0$.

- If $k-\lambda_1$ is a square and $k^2-\lambda_2 v$ is a square such that $k^2-\lambda_2 v \neq 0$, then the multiplicities may not be uniquely determined.
- If the multiplicities are uniquely determined, we admit tuples $(v,k,\lambda_1,\lambda_2,m,n)$ for which the condition $0 = k + (f_1 - f_2)\sqrt{k-\lambda_1} + (g_1 - g_2)\sqrt{k^2-\lambda_2 v}$ holds.
- If the multiplicities are not uniquely determined, we enumerate all possibilities for f_1, try to recover f_2, g_1, g_2 from the conditions $f_1 + f_2 = m(n-1)$, $g_1 + g_2 = m - 1$ and $0 = k + (f_1 - f_2)\sqrt{k-\lambda_1} + (g_1 - g_2)\sqrt{k^2-\lambda_2 v}$. If these conditions are satisfied and there is at most one 0 among f_1, f_2, g_1, g_2, we admit the tuple.

References

[1] D. Crnković, W.H. Haemers, *Walk-regular divisible design graphs*, Designs, Codes and Cryptography, 72 (2014) 165–175.

[2] S. Goryainov, W.H. Haemers, V.V. Kabanov, L. Shalaginov, *Deza graphs with parameters* $(n,k,k-1,a)$ *and* $\beta = 1$, Journal of Combinatorial Designs, 27 (2019) 188–202.

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